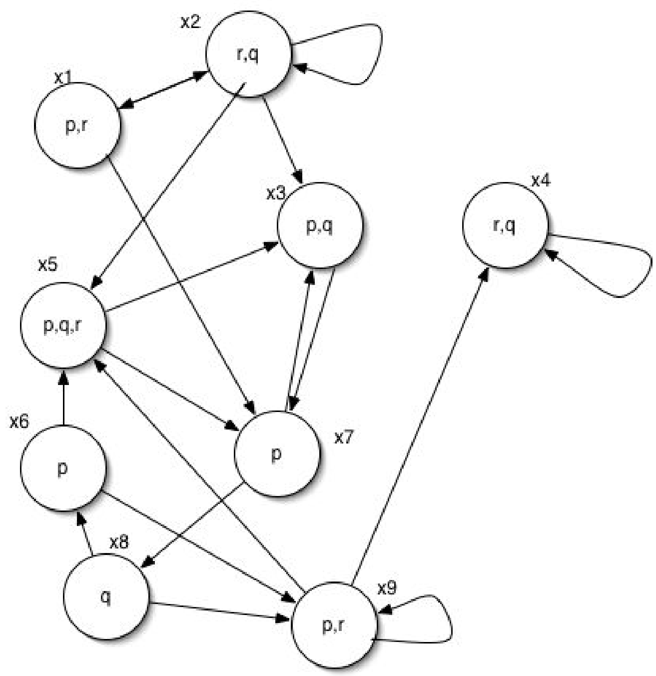
Final Exam Sample Problems with Answers: Parts I and II

Fall 2013, cs112 – James Pustejovsky

[[I will answer some of these, not all. You'll get the hang of it.]]

1 Propositional Modal Logic



1. Find all worlds satisfying:
   (a) $x \Vdash \Diamond(p \wedge q)$;
      i. x2, x5, x5, x7, x9
   (b) $x \Vdash \Box(p \vee r)$;
      i. Check the valuations on each world. Since p or r does not live in every world, this proposition is not true in all worlds, and is not valid in the model. Notice that from x7, the accessible world x8 has neither p nor r. This is enough to invalidate it.
2. Does $x_1 \Vdash \Diamond\Box q$? Show why or why not.
   (a) Find one world, w', such that $x_1 R w'$, where $\Box q$ is true. This means that all worlds, w" from w', $w' R w$", will have q as true, w" $\models q$. It might look like $x2$ is a good candidate,

but notice that $x1$ doesn't cooperate. Now check $x7$. From $x7$, we have $x3$ and $x8$, both of which have q as true. So, the answer is "yes".

3. Does $x_7 \Vdash \Box\Box\Diamond p$? Show why or why not.

   (a) For every world, w', such that x_7Rw', $w' \models \Box\Diamond p$. This means that all worlds, w" from w', $w'Rw$", will have $\Diamond p$ as true, w" $\models \Diamond p$. This means that there is a world w''', w"Rw''', such that $w''' \models p$. So now lets work it out. From $x7$, all worlds are $\{x3, x8\}$. From $x3$, all worlds is only $x7$ again, while for $x8$ all worlds includes $x6$ and $x9$. So, from these worlds, $\Diamond p$ needs to be true, for each one. Examination of the model shows this to be true. Hence, the statement is true.

4. Does $x_9 \Vdash \Diamond(r \vee \Diamond p)$? Show why or why not.

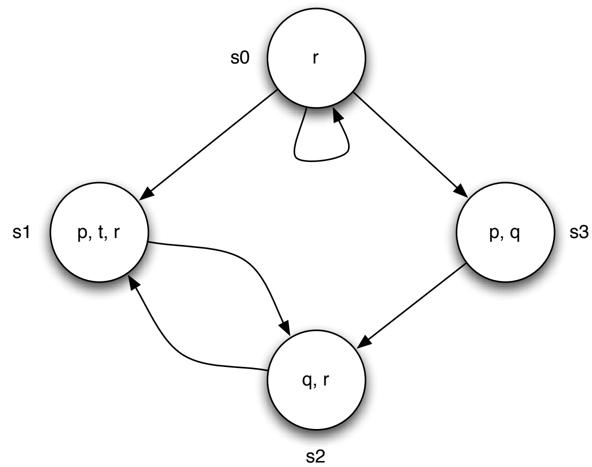
5. Decide whether the following formulas are valid in the model:

   (a) $\Diamond p \vee \Diamond q$

   (b) $\Box(r \wedge \Diamond p)$

2 Linear Temporal Logic

Consider the system M, shown below:



Determine whether $M, s_0 \models \phi$ and $M, s_2 \models \phi$ hold and justify your answer where ϕ is the LTL formula:

(a) $\neg p \to r$

1. $s0 \models \neg p$ and $s0 \models r$, hence this is is true at $s0$. Similarly for $s2$.

(b) Ft

1. $s0 \models Ft$ because there is an s', $s0Rs'$, where $s' \models t$.
   $s2 \models Ft$ because there is an s', $s2Rs'$, where $s' \models t$.

(c) $G(r \vee q)$

1. $s0 \models G(r \vee q)$ because for every s', $s0Rs'$, $s' \models G(r \vee q)$. The same holds for $s2$.

(d) $X(F(t \vee q))$

3 Computation Tree Logic

The following questions use computational tree logic. For your convenience, we have given you what each of the operators mean:

A: along all paths

E: along at least one path

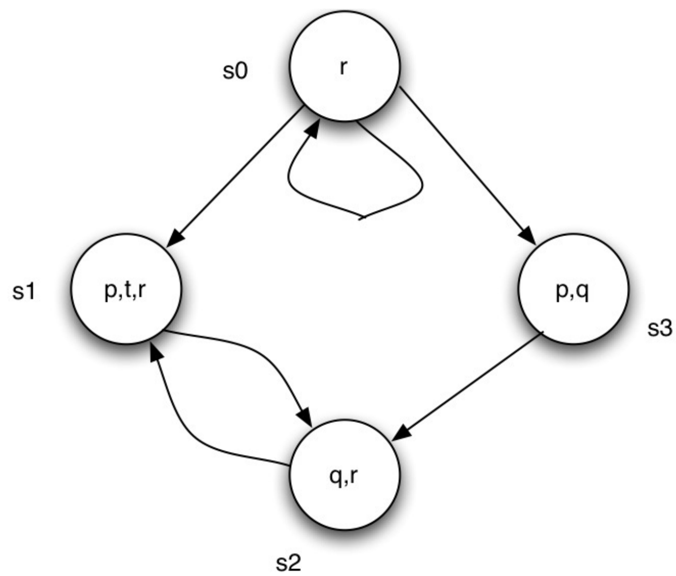
X: next state

F: some future state

G: all future states including the current state

U: until

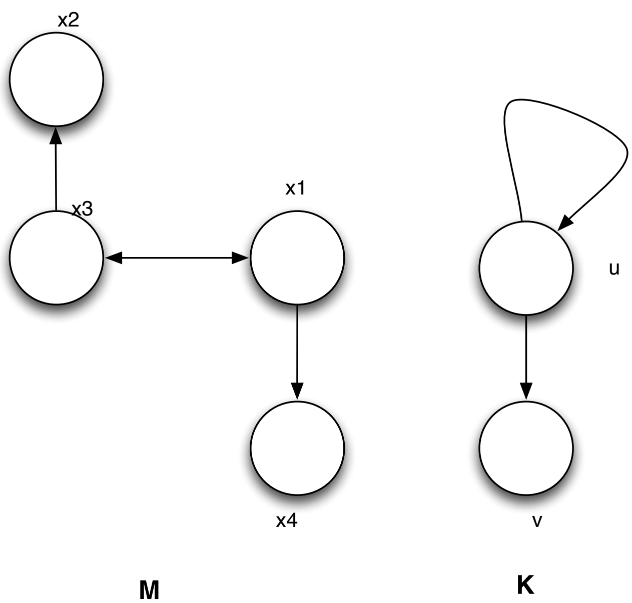
Use the following model to answer the questions below:



1. Does $s0 \Vdash AFt$? Show why or why not.
   (a) No. t eventually shows up on almost all paths from $s0$; immediately for $s1$; in 3 steps for $s3, s2, s1$; and in 4 steps for $s0, s3, s2, s1$. But if you keep looping in $s0$, $s0, s0, \ldots s0$ you will never encounter t. Hence, it is not true.
2. Does $s0 \Vdash \neg EGr$? Show why or why not.
3. Does $s0 \Vdash E(tUq)$? Show why or why not.
4. Does $s0 \Vdash AG(r \vee q)$? Show why or why not.
5. Decide whether the following formula is valid in the model: AFq
   (a) Yes. For any state, s, all states s', sRs', $s' \models Fq$.

4 Bisimulation

Consider the two models, **M** and **K** below.



Apply the bisimulation algorithm to see if these models are in fact, bisimilar.

1. Answer: Yes.