

CS112 Notes: Semantics of First-order Logic

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A model M for a language L of predicate logic consists of a domain D and an interpretation function, I . When we view the interpretation function as relative to an assignment, g , of specific values to variables, then we speak of an interpretation function, $\llbracket \cdot \rrbracket_{M,g}$, in a model M under an assignment g .

Definition 1:

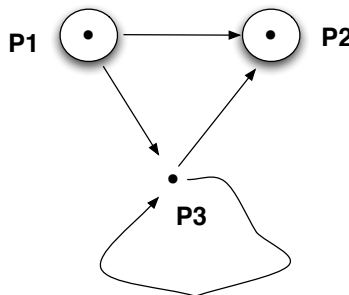
- (i) $\llbracket t \rrbracket_{M,g} = I(t)$ if t is a constant in L ;
- (ii) $\llbracket t \rrbracket_{M,g} = g(t)$ if t is a variable.

We can now give the full definition of expressions in the predicate calculus, within a model and assignment function:

Definition 2:

- (i) If $A(a_1, \dots, a_n)$ is an atomic sentence in L , then $V_{M,g}(A(t_1, \dots, t_n)) = 1$ iff $\langle \llbracket t_1 \rrbracket_{M,g}, \dots, \llbracket t_n \rrbracket_{M,g} \rangle \in I(A)$.
- (ii) $\llbracket \neg\phi \rrbracket_{M,g} = 1$ iff $V_{M,g}(\phi) = 0$
- (iii) $\llbracket \phi \wedge \psi \rrbracket_{M,g} = 1$ iff $V_{M,g}(\phi) = 1$ and $V_{M,g}(\psi) = 1$
- (iv) $\llbracket \phi \vee \psi \rrbracket_{M,g} = 1$ iff $V_{M,g}(\phi) = 1$ or $V_{M,g}(\psi) = 1$
- (v) $\llbracket \phi \rightarrow \psi \rrbracket_{M,g} = 1$ iff $V_{M,g}(\phi) = 0$ or $V_{M,g}(\psi) = 1$
- (vi) $\llbracket \phi \leftrightarrow \psi \rrbracket_{M,g} = 1$ iff $V_{M,g}(\phi) = V_{M,g}(\psi)$.
- (vii) $\llbracket \forall v\phi \rrbracket_{M,g} = 1$ iff for all $d \in \mathcal{D}$: $V_{M,g[v/d]}(\phi) = 1$
- (viii) $\llbracket \exists v\phi \rrbracket_{M,g} = 1$ iff there is at least one $d \in \mathcal{D}$: $V_{M,g[v/d]}(\phi) = 1$

Assume a model \mathcal{M} , where constants and predicates have the following interpretation:



Model M_1 :

$$I(a) = P_1, I(b) = P_2, I(c) = P_3, I(A) = \{P_1, P_2\}$$
$$I(R) = \{\langle P_1, P_2 \rangle, \langle P_1, P_3 \rangle, \langle P_3, P_2 \rangle, \langle P_3, P_3 \rangle\}$$

Now let us use this definition to determine the truth value of the following sentences:

$$\exists x \exists y \exists z [R(x, y) \wedge A(y) \wedge R(x, z) \wedge \neg A(z)]$$

What are the steps in calculating the interpretation of this expression for the above model M_1 ?